Multilevel Approach to Minimum Weight Design including Buckling Constraints

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A rational multilevel approach for minimum weight structural design of truss and wing structures including local and system buckling constraints is presented. Overall proportioning of the structure is achieved at the system level subject to strength, displacement, and system buckling constraints while the detailed component designs are carried out separately at the component level satisfying local buckling constraints. Total structural weight is taken to be the objective function at the system level while employing the change in the equivalent system stiffness of the component as the component level objective function. Approximation concepts including design variable linking, constraint deletion, and explicit constraint approximations are employed at the system level. Finite-element analysis is used to predict static response while system buckling behavior is handled by incorporating a geometric stiffness matrix capability. Buckling load factors and the corresponding mode shapes are obtained by solving the eigenvalue problem associated with the assembled elastic stiffness and geometric stiffness matrices for the structural system. At the component level various local buckling failure modes are guarded against using semiempirical formulas. Mathematical programming techniques are employed at both the system and component level, and the information transferred between the levels is carefully selected so as to enhance overall convergence of the entire design procedure.

Introduction

THIS paper presents a multilevel method that includes consideration of local and system buckling constraints in the context of a modern finite-element analysis/synthesis capability. These buckling constraints are to be considered in addition to the usual stress, displacement, and member size limitations treated in Refs. 1-4. During the past four years the use of approximation concepts has produced dramatic improvements in the efficiency of structural optimization capabilities based on combining finite-element analysis methods and mathematical programming techniques. In particular, design variable linking, temporary deletion of inactive and/or redundant constraints, and the generation of high-quality explicit approximations for retained constraints have played a central role in these recent advances.

Very little work has been done on including buckling constraints in finite-element analysis/synthesis capabilities. Discretized optimality criteria methods that treat system buckling and minimum size constraints have been reported.[5,6] However, these studies fail to treat system buckling in parallel with other common behavorial constraints such as static stress and displacement limitations. Furthermore, they completely ignore the more difficult problems associated with guarding against local buckling failure modes. On the other hand, Refs. 7 and 8 include consideration of local buckling failure modes in the design of wing and fuselage structures, respectively. In fact, Refs. 7 and 8 employ a multilevel approach; however, the method presented suffers from two primary shortcomings: 1) the use of weight as the objective function at the component level and 2) the use of fully stressed-type resizing algorithms at the system level.

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Basic Approach

Addition of system buckling within the context of previous studies[1-4] is rather straightforward since the structural analysis is based on the displacement method. System buckling constraints can be incorporated by adding geometric stiffness matrix capability and solving the associated eigenproblems for buckling load factors and the corresponding mode shapes. Mathematically this type of failure mode has many characteristics similar to those involved when dealing with frequency constraints (e.g., Ref. 4).

On the other hand, the inclusion of local buckling constraints presents difficulties because their meaningful representation requires that some consideration be given to the detail design of the many individual components which make up the structural system. Inclusion of all of the detailed variables of the components in a single large methematical programming problem statement rapidly increases the number of design variables and a direct attack on the problem in this form is impractical.

Independent of any reservations one may have about its detailed implementation in Refs. 7 and 8, the multilevel concept appears to be a sound basic approach for coping with local buckling constraints. Therefore, a multilevel approach is adopted in this paper. The new multilevel approach presented herein overcomes the previously noted shortcomings by 1) employing stiffness change as the component level objective function to be minimized and 2) using approximation concepts to facilitate rational design improvement at the system level.

The key new idea in the multilevel approach presented here is to select the component level objective function so as to minimize disturbance of component level loading due to component level synthesis. This is accomplished by taking the change in stiffness as the component level objective function to be minimized. Note that in Refs. 7 and 8 weight is taken as the component level objective function, even though it is recognized that a structure made up of minimum weight components is not necessarily a minimum weight system. In the work reported here total system weight is taken as the system level objective function, and force redistribution is controlled by using relatively tight move limits during system level design improvement. In the multilevel method set forth here, mathematical programming formulations are employed

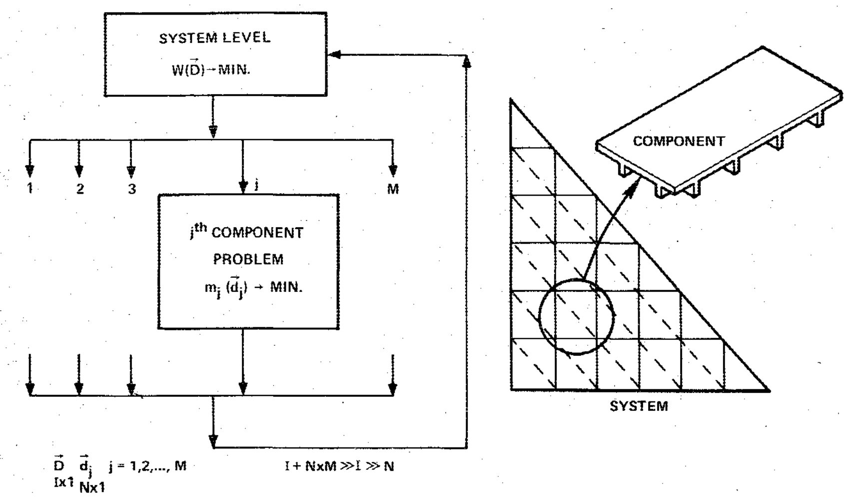

Fig. 1 Multilevel approach for buckling constraints.

at both the component and system levels. The use of mathematical programming techniques at the system level is made possible by recent developments in approximation concepts for structural synthesis.[1-4] In a sense, the work reported here may be viewed as an extension of ACCESS 1[2,3] to include local and system buckling constraints in parallel with stress, displacement, and member size constraints.

The multilevel approach is outlined schematically in Fig. 1. The multilevel scheme employed here may be viewed as a decomposition method in which the formulation is guided by physical insight. The formulation decomposes the primary problem statement into a system level design problem and a set of uncoupled component level problems. Results are obtained by iterating between the system and component level problems. Convergence of the multilevel process is enhanced by selecting a component level objective function so as to weaken the coupling between the system and component level problems. The procedure used in this paper begins with the determination of system level allowable stresses based on the initial component level design. Then, one stage of design improvement is carried out at the system level using these allowable stresses. The load distribution for the improved design is determined by a complete system level structural analysis, and this component loading information is supplied to the component design optimization subproblems. Each component is then modified so that all of the component level constraints are satisfied, particularly local buckling, while striving to force the component stiffness to conform to the value supplied by the foregoing system level design improvement stage. After all of the components are redesigned, the allowable stresses are updated and the next stage of design improvement at the system level follows.

Formulation

The general problem can be stated as follows: Find X such that

$$W(X) \rightarrow \text{min.} \tag{1}$$

and

$$G_p(X) \geq 0 \quad p \epsilon \tilde{P} \tag{2}$$

where W, the total system weight, is the objective function and $\tilde{P}$ represents the complete set of constraints for the structural design problem at hand. Let the set of system design variables be contained in $\boldsymbol{D}$. Let the detailed design variables for the jth component be contained in $\boldsymbol{d}_j$, and let $\boldsymbol{d}$ denote the concatenation of the $\boldsymbol{d}_j$ for all M components. Then the constraints G_p can be broken up into two categories. The first category contains constraints that are strongly dependent on the $\boldsymbol{D}$, such as system buckling, while the second category contains constraints that are primarily dependent on the component variables $\boldsymbol{d}_j$. Now the primary problem stated by Eqs. (1) and (2) can be rewritten as:

$$\text{find } \boldsymbol{D} \text{ and } \boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3, \ldots \boldsymbol{d}_M \text{ such that} \tag{3}$$

$$G_q(\boldsymbol{D}, \boldsymbol{d}) \geq 0; \quad q \epsilon Q \tag{4}$$

and

$$g_{lj}(\boldsymbol{d}_j, \boldsymbol{D}) \geq 0; \quad l \epsilon L; \quad j \epsilon M \tag{5}$$

and

$$W(\boldsymbol{D}) \rightarrow \text{min.} \tag{6}$$

where Q denotes the set of system level constraints, L denotes the set of local constraints applicable to the jth component problem, M represents the number of components, and it is understood that

$$\boldsymbol{d}^T = [\boldsymbol{d}_1{}^T, \boldsymbol{d}_2{}^T, \boldsymbol{d}_3{}^T, \ldots, \boldsymbol{d}_M{}^T]$$

Now, recasting Eqs. (3-6) as a multilevel synthesis problem with approximation concepts at the system level[2,3] gives the following.

1) System level: Find $\boldsymbol{D}$ such that

$$W(\boldsymbol{D}) \rightarrow \text{min.} \tag{7}$$

and

$$\tilde{G}_q(\boldsymbol{D}, \boldsymbol{d}^*) \geq 0; \quad q \epsilon Q_R \tag{8}$$

where $\boldsymbol{d}^*$ implies that none of the detailed design variables $\boldsymbol{d}$ change during a system level design modification stage, $\tilde{G}_q$ indicates the use of explicit approximations for the system level constraints, and Q_R represents the reduced set of system level constraints retained after deletion of those that are neither critical nor potentially critical.

2) Component level for each component ($j = 1,2,\ldots M$): Find $\boldsymbol{d}_j$ such that

$$m_j(\boldsymbol{d}_j) \rightarrow \text{min.} \tag{9}$$

and

$$g_{lj}(\boldsymbol{d}_j, \boldsymbol{D}^*) \geq 0; \quad l \epsilon L \tag{10}$$

where $\boldsymbol{D}^*$ implies that the system level design variables are held invariant during the component design modification stage. As mentioned before, the component objective function is chosen so as to minimize the change in component stiffness. In general the stiffness characteristics of the jth component depend on a set of R stiffness parameters. These stiffness characteristics can be written as $K_{rj}(\boldsymbol{D}_j)$ and they

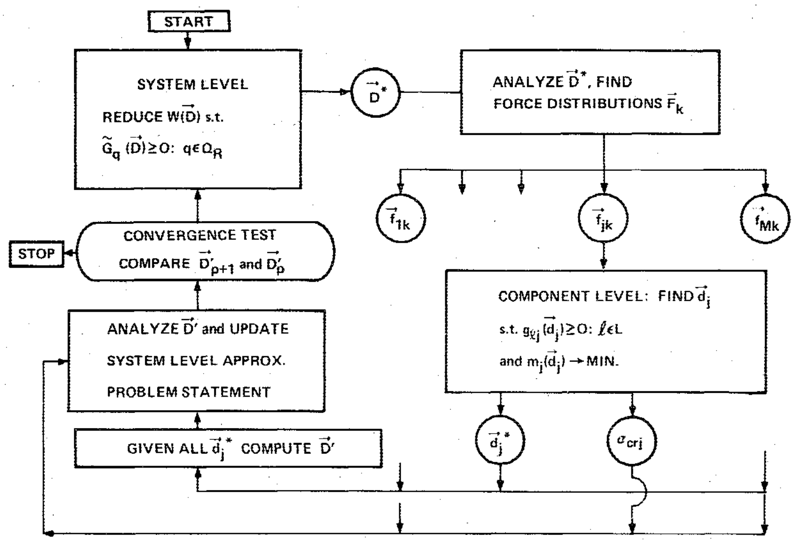

Fig. 2 Implementation of the multilevel approach.

depend on the subset of system design variables associated with the jth component (D_j) as obtained at the end of a system design modification stage. In general it is also possible to express each of the R stiffness parameters for the jth component as a function $H_{rj}(d_j)$ of the component design variables d_j. Now the objective function for the jth component during a stage in the multilevel synthesis can be written as

$$m_j = \sum_{r=1}^{R} \left[K_{rj}(D_j^*) - H_{rj}(d_j)\right]^2 \qquad (11)$$

where D_j^* is the value of the system design variables corresponding to the jth component at the end of the foregoing system stage.

The flow chart in Fig. 2 further elucidates the multilevel approach as implemented in this study.

Optimization Procedure

A sequence of unconstrained minimization techniques based on the extended interior penalty function formulation[9] and a modified Newton method[10] for carrying out the unconstrained minimizations is used in this work. This algorithm is called NEWSUMT and it is described in considerable detail in Ref. 2.

The component design modification stage consists of a complete component synthesis for each component. During a stage in the component synthesis exact function evaluations are used. Furthermore, all of the constraints are retained. It was decided not to use constraint deletion or Taylor series expansions at the component level since each component problem involves a relatively small number of constraints and they are explicit functions of only a few local design variables d_j. It should, however, be recognized that simplified buckling formulas used at the component level do represent an approximate analysis.

At the system level all of the approximation concepts introduced in Refs. 2 and 3 are employed: namely, design variable linking, temporary deletion of redundant constraints, and the generation of high-quality explicit approximations for retained constraints.

Applications

The previously described multilevel approach has been applied to tubular truss and integrally stiffened wing box structures. The ACCESS 1 computer code[2] was modified to incorporate the multilevel approach and the necessary constraints. This also illustrates the usefulness of the ACCESS 1 code as a research tool pointing up the ease with which it can be modified and extended.

In practice, components belonging to the same region may be required to have the same design. This leads to the use of design variable linking at the system level while reducing the number of component subproblems at the component level.

Thin-Walled Tubular Structures

For this class of truss structures the system level design variables (D) are the cross-sectional areas (A) after linking, and the design variables (d_j) for the jth component are mean diameter B_j and wall thickness T_j. Design variable linking at the system level is limited to members of identical length. The two types of local buckling failure that the truss members may experience are Euler buckling and member crippling. Using the optimum stress concept of Ref. 11 it is possible, in the absence of side constraints, to reduce the number of component variables to one for each member. However, this usually leads to large and hence impractical B_j/T_j ratios. Therefore, since side constraints are usually needed in practice (e.g., minimum gage on T_j and maximum diameter on B_j), the multilevel approach is appropriate.

The force-strain relationship for truss elements can be characterized by a single term since they carry only uniaxial loads. Consequently, the single stiffness parameter in terms of the design variables at the two levels can be written as

$$K_{lj} = E_j A_j \qquad (12)$$

$$H_{lj} = E_j \pi B_j T_j \qquad (13)$$

where E_j is the modulus of elasticity for the number.

The component objective function m_j hence reduces to

$$m_j(d_j) = (K_{lj}^* - H_{lj})^2 = (E_j A_j^* - E_j \pi B_j T_j)^2 \qquad (14)$$

where the superscript * implies that the associated quantities are held invariant during the component modification stage. The fixed member area A_j^* is the value obtained at the end of the foregoing system design modification stage.

Now, the minimum weight multilevel design problem for truss structures can be stated as follows:

1) System level: Find A such that

$$W = \sum_{i=1}^{I_T} \rho_i A_i l_i \rightarrow \min. \qquad (15)$$

and the following constraints are satisfied

Side

$$A_i^U \geq A_i \geq A_i^L \quad i \in I_T \tag{16}$$

Nodal displacements

$$u_i^U \geq u_{ik} \geq u_i^L \quad i \in I_u; \quad k=1,2,...K \tag{17}$$

System buckling

$$\lambda_{lk} \geq \gamma_{lk} \, l \in L, \quad k=1,2,...K \tag{18}$$

Element stresses

$$\sigma_i^U \geq \sigma_{ik} \geq \sigma_i^m \quad i \in I_T, \quad k=1,2,...K \tag{19}$$

where

$$\sigma_i^m = -\text{absolute min. of } \{\sigma^L, \ [\sigma^E(d_j^*)]_{cr}, \ [\sigma^{cp}(d_j^*)]_{cr}\} \tag{20}$$

2) Component level for each distinct component ($j=1,2,...M$): Find B_j, T_j such that

$$m_j(d_j^*) = (EA_j^* - E_j \pi B_j T_j)^2 \rightarrow \text{min.} \tag{21}$$

and the following constraints are satisfied

Side

$$B_j^U \geq B_j \geq B_j^L \tag{22}$$

$$T_j^U \geq T_j \geq T_j^L \tag{23}$$

$$(B/T)_j^U \geq (B_j/T_j) \geq (B/T)_j^L \tag{24}$$

Stress

$$P_{cj}^* - \sigma_j^L \pi B_j T_j \geq 0 \tag{25}$$

$$\sigma_j^U \pi B_j T_j - P_{tj}^* \geq 0 \tag{26}$$

Euler buckling

$$\frac{P_{cj}^*}{\pi B_j T_j} + \frac{E_j \pi^2 B_j^2}{8 l_j^2} \geq 0 \tag{27}$$

Member crippling

$$\frac{P_{cj}^*}{\pi B_j T_j} + \frac{K_{cj} E_j T_j}{B_j} \geq 0 \tag{28}$$

where the local Euler buckling stress, $(\sigma_j^E)_{cr}$ and member crippling stress $(\sigma_j^{cp})_{cr}$ are given by

$$(\sigma_j^E)_{cr} \approx E_j \frac{\pi^2 B_j^2}{8 l_j^2} \text{ for } B_j/T_j \geq 15 \tag{29}$$

and

$$(\sigma_j^{cp})_{cr} = K_{cj} E_j T_j / B_j \tag{30}$$

In the foregoing equations, ρ and l are, respectively, the density and length of the TRUSS element; I_T is the total number of TRUSS elements; the supercripts U and L denote the upper and lower limits of the associated quantities; K is an index representing the number of load conditions; I_u represents the number of constrained nodal displacements; and u denotes a nodal displacement component. In the system buckling constraint [Eq. (18)], the eigenvalue (or load factor) associated with the lth mode shape and the kth load condition is represented by λ_{lk} while γ_{lk} denotes the corresponding required factor of safety. For each load condition k the first L buckling load factors (λ_{lk}) and the associated buckling mode shapes $(\{\phi\}_{lk})$ are generated. The eigenproblem from which this information is extracted is set up by assembling the system stiffness and geometric stiffness matrices from the corresponding element matrices (expressed in the system reference coordinates). The resulting eigenproblem has the familiar form

$$[K]\{\phi\}_k = \lambda_k [K_{Gk}]\{\phi\}_k \qquad k \in K \tag{31}$$

Note that $[K]$ the system stiffness matrix is a function of the system level design variables (A) and $[K_{Gk}]$, the system geometric stiffness matrix depends on both the system design variables A and the load condition.

The quantities $(\sigma^E(d_j^*))_{cr}$ and $(\sigma^{cp}(d_j^*))_{cr}$ [see Eq. (20)] are the Euler buckling stress and member crippling stress based on the component design d_j at the end of the foregoing component design modification stage, while σ^L is a fixed lower stress limit. Hence, σ_i^m is invariant during each system stage, but it is updated at the end of each component design modification stage. The crippling stress coefficient K_{cj} [see Eq. (20)] is usually taken to be 0.4. The loads for the component level (P_{tj}^* and P_{cj}^*) are the maximum and minimum forces for the component design group, obtained from the system analysis performed at the end of the foregoing system design modification stage. It should be noted that Eq. (24) places bounds on the mean diameter to wall thickness ratio (B/T) that are independent of the member size constraints embodied in Eqs. (22) and (23).

Wing Structures

The multilevel approach has also been applied to the design of the wing box structures. For the sake of simplicity, only

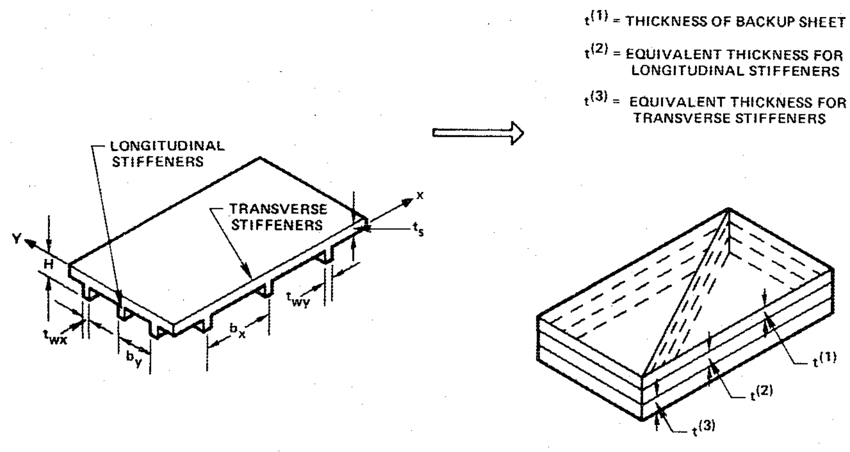

Fig. 3 **System idealization of the component.**

midplane symmetric wings are considered. Spar and rib caps (if any) are modelled using TRUSS elements while webs are represented by symmetric shear panel (SSP) elements.[2] The wing cover plates, also referred to as skin panels, are made up of integrally stiffened waffle plates. These waffle plates have spanwise as well as chordwise stiffening. To complete the system level idealization of the wingbox, the other elements being TRUSS and SSP elements, each skin panel component is divided into two triangular regions (Fig. 3), and each region contains a merged stack of three orthotropic constant strain triangular elements (CSTOR). In this merged stack, the first level of CSTOR elements represents the backup sheet, the second represents equivalent thickness of the longitudinal stiffeners, while the third and last level represents the equivalent thickness of the transverse stiffeners. The use of three independent, equivalent thickness system level design variables per waffle plate component is important because it permits the modification of the component's orthotropy to take place during each system level modification stage. It is emphasized that changes in panel orthotropy can play a key role in redistributing internal loads, therefore design variables controlling gross panel orthotropy (e.g., three equivalent thicknesses per component) should be subject to change at the system level in the design process.

At the system level then, the design variables (D) include the cross-sectional areas A_i of TRUSS elements, thickness of SSP elements τ_i, and thickness of CSTOR elements t_i, stacked to simulate the gross stiffness characteristics of waffle plate components (Fig. 3). It is assumed that the skin panels are the only part of the wing box requiring detailed design. Each independent waffle plate component is described by the following detailed design variables d_j (Fig. 3): thickness of the backup sheet (t_s) spacing and thickness of longitudinal stiffeners (b_y and t_{wx}, respectively), spacing and thickness of transverse stiffeners (b_x and t_{wy}, respectively), and overall depth of the panel H. This gives a total of six detailed design variables for each independent waffle plate component. It should be noted that several waffle plate components can be linked into a component group. When this is done, all of the components in the group are identical, and there will be three system level design variables (equivalent thicknesses) and six component level design variables describing the set of components in the group (Fig. 6).

The primary loads carried by the skin panels of the wing box structures are inplane shear N_{xy} and biaxial force resultants N_x, N_y in the spanwise and chordwise directions, respectively. Assuming orthotropic skin panel components, the force resultants N_x, N_y, and N_{xy} are related to the inplane strains ϵ_x, ϵ_y, and γ_{xy} by the familiar relationship

$$\{N\} = [A]\{\epsilon\} \tag{32}$$

which involves four independent membrane stiffness properties, namely, A_{11}, $A_{12} = A_{21}$, A_{22}, and A_{66}. These stiffness properties when expressed explicitly in terms of the system design variables (equivalent thicknesses $t_j^{(1)}$, $t_j^{(2)}$, and $t_j^{(3)}$ for the jth component as shown in Fig. 3) are denoted as K_{1j}, K_{2j}, K_{3j}, and K_{4j}. On the other hand, the membrane stiffness properties A_{mn}, when expressed explicitly in terms of the component level design variables, are denoted by H_{1j}, H_{2j}, H_{3j}, and H_{4j}. Representative samples of the explicit expressions for K_{rj} and H_{rj} are given in the following

$$K_{1j} = A_{11}(t_j) = \frac{E_j t_j^{(1)}}{(1-\nu_j^2)} + E_j t_j^{(2)} \tag{33}$$

$$H_{1j} = A_{11}(d_j) = \frac{E t_s}{(1-\nu^2)} + E(H - t_s)\frac{t_{wx}}{b_y} \tag{34‡}$$

‡For convenience the subscripts j have been omitted in writing the detailed design variables and the elastic properties on the right-hand side of Eq. (34).

and the complete set of relations will be found in Chap. 6 of Ref. 12.

Now denoting the stiffness of the component at the end of the foregoing system level design modification stage to be K_{rj}^*, the component level objective function becomes [see Eq. (11)]

$$m_j = \sum_{r=1}^{4} \left(K_{rj}^* - H_{rj}\right)^2 \tag{35}$$

Replacing K_{rj}^* by A_{mn}^* and H_{rj} by A_{mn} in Eq. (35) yields

$$m_j(d_j) = [A_{11}^* - A_{11}]^2 + [A_{12}^* - A_{12}]^2 + [A_{22}^* - A_{22}]^2 + [A_{66}^* - A_{66}]^2 \tag{36}$$

In Eq. (36), $A_{mn}^* = A_{mn}(t_j^*)$ and these constants can be evaluated by substituting t_j^* (the value of the equivalent thicknesses, for component j, at the end of the foregoing system level design modification stage) into relations like that given by Eq. (33). Also, in Eq. (36), the $A_{mn} = A_{mn}$ (d_j) are explicit algebraic expressions in terms of the detailed design variables for the jth component, such as that given by Eqs. (34) for A_{11} (d_j).

Having established the component level objective function, the next step in formulating the problem is to specify the constraints to be imposed. At the component level, in addition to bounds on the detailed design variables d_j, constraints devised to guard against gross panel buckling, stiffener buckling, and sheet instability are also imposed. The component level failure modes are based on approximate buckling analyses given in Ref. 12 which broadly speaking follow the approach used earlier in Ref. 13. Gross panel buckling is guarded against using an interaction formula approach (see Chap. 6, Ref. 12) which compares applied component loads (N_x, N_y, N_{xy}) with gross panel buckling loads [$(N_x)_{cr}$, $(N_y)_{cr}$, and $(N_{xy})_{cr}$]. The component applied loads are taken to be the average of the forces in the two triangular regions (each triangular region generally contains a merged stack of three CSTOR elements) representing the panel at the system level. During each component design modification stage the gross panel applied loads are assumed to be invariant. This is a good assumption because the component level objective function employed herein tends to reduce system level force redistribution, during a component level design modification stage, to the vanishing point. On the other hand, the gross panel buckling loads are nonlinear but explicit functions of the detailed design variables describing the jth component. For stiffener and sheet instability constraints, the loads carried by them are compared with the corresponding buckling loads. Although the total forces carried by the component are considered invariant, the forces carried by the stiffeners and the backup sheet are allowed to vary relative to one another. Since stiffener buckling and sheet instability are localized failure modes, the forces in both triangular regions of the panel are given due consideration.

At the system level stress, displacement, minimum and maximum size constraints, as well as system and local buckling constraints, are imposed. The system level local buckling constraints are formed using invariant critical buckling loads based on the detailed panel designs available at the end of the previous component level design modification stage. However, at the system level the panel applied loadings are functions of the system level design variables. That is to say, redistribution of internal forces generally occurs during each system level design modification stage. It should be noted that move limits are used during each system level design modification stage so as to prevent drastic force redistributions that could adversely affect overall convergence of the multilevel design procedure. It should be recognized that the local buckling constraints at the component level differ from those at the system level in that the total component applied loads are held constant while the critical

Table 1: CPU run times for the examples[a]

Example No. / Column No.	CPU run times, s: Total for component level	Total for system level	Total ②+③	Number of stages
①	②	③	④	⑤
1) Truss tower				
Case 1	39.4	18.4	57.8	11
Case 2	63.7	33.6	97.3	19
Case 3	51.2	26.4	77.6	16
Case 4	...	14.5	14.5	10
2) Delta wing				
Run 1	1563.5	164.5	1738.0	25
Run 2	1299.0	162.7	1461.7	21
Run 3	...	59.8	59.8	13

[a]Runs were made on the IBM 360/91 computer at UCLA using FORTRAN H compiler.

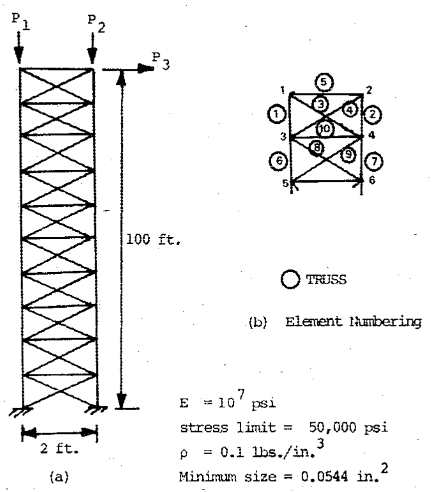

Fig. 4 Truss tower.

buckling loads are functions of the detailed design variables. The mathematical formulation of the wing box problem with detailed explanations can be found in Chap. 6 of Ref. 12.

Examples

The multilevel approach presented here has been applied to the design of several tubular truss and wing box structures. Computational efficiency, as measured by run times, is sensitive to the termination criterion used at the component level. Run times for the examples shown here are given in Table 1. Several illustrative examples are briefly discussed here, and a more extensive collection of detailed results can be found in Ref. 12.

For the wing box examples presented here, attention is focused on seeking the minimum weight design for the top cover of these midplane symmetric structures under the specified load conditions. During the optimum design procedure it is assumed that none of the constraints related to the bottom half of the structure are critical. This assumption was verified for the delta wing by analyzing the top half final design of the wing subject to a single reversed load condition.

Example 1: Truss Tower

A minimum weight structure designed against failure in local Euler buckling, member crippling, system buckling (load factor of unity), minimum member area of 0.0544 in.^{2} as well as upper and lower stress limits of 50,000 psi is sought for a 50-bar planar truss tower (Fig. 4). The structure is subjected to two load conditions namely: a) $P_1 = P_2 = 5$ kips, $P_3 = 0$, and b) $P_1 = P_2 = 0$, and $P_3 = 2$ kips (see Fig. 4). Two other cases (2 and 3) were created by adding more constraints to the set just described. Case 2 was formed by adding the following practical limits on the detailed design variables: a) $B^U = 4$ in., b) $T^L = 0.02$ in. and $(B/T)^U = 50$. Case 3 was generated by adding a displacement constraint to the set of constraints considered in case 2. In particular the lateral displacements of the nodes at the top of the tower (i.e. nodes 1 and 2, see Fig. 4) were limited to a maximum value of 75 in.

In case 1 the minimum weight of the tower comes out to 488.03 lb. The wall thickness for the members range from 0.013 to 0.102 in. while the mean diameters range from 4.55 to 7.8 in. These design values for B and T result in (B/T) ratios that range from a high of 228.7 to a low of 76.7. The resulting design is critical with respect to system buckling and simultaneous local Euler and member crippling in the compression diagonals. As for case 2 the minimum weight achieved turns out to be 1035.8 lb, more than twice the minimum weight obtained in case 1. The critical constraints at the optimum design for case 2 are a) lower limits on the areas of the horizontal members; b) upper limits on the diameters and Euler buckling for the vertical members in compression; and c) upper limits and Euler buckling in the compression diagonals. Because of the limitation on the maximum diameter, the local Euler buckling stress is quite small when compared with the fixed lower stress limit. The second load condition is primarily responsible for the critical Euler buckling stress constraints. In case 3 the minimum weight achieved is 1185.8 lb. Comparing this to the minimum weight achieved in case 2 (i.e., 1035.8 lb) indicates that the weight penalty associated with satisfying the added displacement constraint is 150 lb. The final design for case 3 is shown in Table 2. When the detailed results for case 3 were compared with those for case 2, it was found that new material has been added only to the vertical members. This was gratifying, since it is intuitively reasonable to expect that adding bending

Table 2: Final designs for truss tower

Element	A, in.^{2}	B, in.	T, in.	Element	A, in.^{2}	B, in.	T, in.
1,2	0.612	3.12	0.062	3,4	0.618	3.13	0.063
6,7	1.284	3.87	0.106	8,9	0.625	3.15	0.063
11,12	2.140	3.87	0.176	13,14	0.624	3.15	0.063
16,17	2.997	3.87	0.246	18,19	0.624	3.15	0.063
21,22	3.853	3.87	0.317	23,24	0.624	3.15	0.063
26,27	4.709	3.87	0.387	28,29	0.624	3.15	0.063
31,32	5.565	3.87	0.457	33,34	0.624	3.15	0.063
36,37	6.42	3.87	0.528	38,39	0.624	3.15	0.063
41,42	7.277	3.87	0.598	43,44	0.624	3.15	0.063
46,47	8.134	3.87	0.669	48,49	0.624	3.15	0.063
Others	0.0545	0.63	0.028				
Total weight	1185.8 lb						

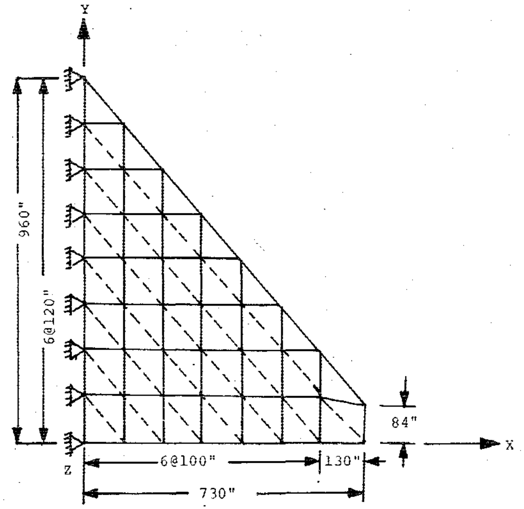

Fig. 5 Delta wing.

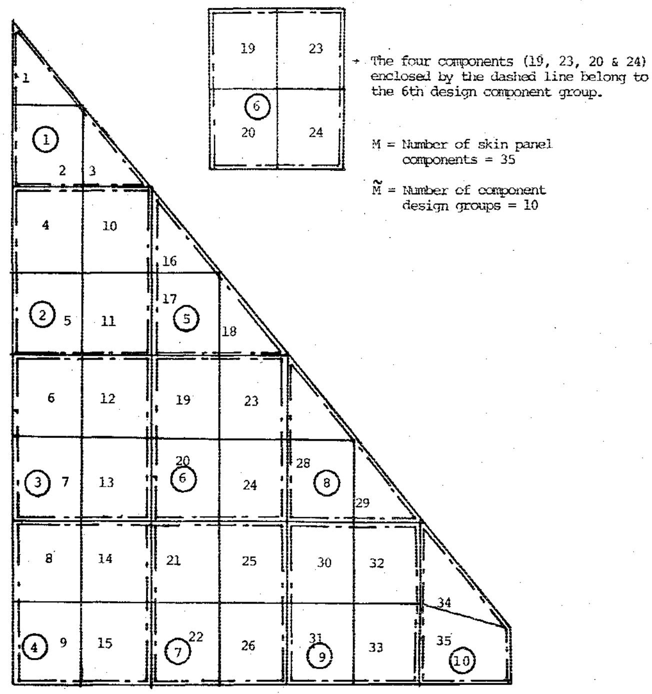

Fig. 6 Linked design components for the delta wing.

stiffness to the tower would be the most effective means of satisfying the lateral displacement constraint. In both cases 2 and 3, Euler buckling was critical in the compression verticals while maximum B/T ratios and Euler buckling were critical in the compression diagonals.

Finally, the truss tower problem statement was modified (case 4) so that it is identical to the problem treated in Ref. 6. In brief, the problem is to design the tower for only the first load condition subjected to minimum area, system buckling (with required load factor of unity), and stress constraints ($\sigma^U = |\sigma^L| = 20{,}000$ psi). The final design was substantially the same as that given in Ref. 6 and had a weight of 468.2 lb (as against a value of 466.6 lb given in Ref. 6).

Example 2: Delta Wing

The multilevel method developed herein has also been applied to the previously studied[2,3] titanium delta wing problem (Fig. 5). The structure is assumed to be symmetric with respect to its middle surface and the upper half is modelled using 189 orthotropic CST elements for the skin panels and 70 symmetric shear panel elements for the vertical webs. The wing is subject to a single static load condition that is roughly equivalent to a uniformly distributed loading of 144 psf.

The delta wing surface is assumed to consist of 35 integrally stiffened panels (Fig. 6). The problem is to determine both the system and the component level design variables so as to minimize the total weight of the structural system. Design variable linking was used at the component level to reduce the number of distinct waffle plates to 10. For example it is understood that the four waffle plate panels in region 6 of Fig. 6 are to be identical. Consistent with the foregoing, design variable linking was also used at the system level leading to 30 system level design variables for the CST elements and 12 design variables for SSP elements, for a total of 42 system level design variables. It should be understood that there are three independent equivalent thickness design variables associated with each of the 10 regions shown in Fig. 6.

The design of the delta wing with the inclusion of local buckling constraints is achieved using two runs. In the first run an arbitrary thin stiffener type of initial design for all components is employed. This results in a final weight of 80,000 lb. A second run was made in which the spanwise stiffener spacings were fixed at $b_y = 4$ in. and the chordwise stiffener spacings were fixed at $b_x = 6$ in. The starting design for run 2 was the same as the final design of run 1 but with the spacings changed to the foregoing fixed values. This required the thicknesses of the stiffeners t_{wx} and t_{wy} to be adjusted so that the starting design of run 2 had the same (t_{wx}/b_y) and (t_{wy}/b_x) ratios as the final design of run 1. The minimum weight achieved in run 2 was 59,600 lb which is about 20,000 lb lighter than the best result obtained in run 1. In the absence of relative minima pockets, the final weight for run 1 should be the same or less than that achieved at the end of run 2. However, it is known that relative minima exist at the

Table 3 Final component designs for delta wing with fixed stiffener spacings[a]—Run 2

Design component group no.	Height of panel H, in.	Thickness of sheet, t_s, in.	Thickness of spanwise stiffeners, t_{wx}, in.	Thickness of chordwise stiffeners, t_{wy}, in.	$\frac{t_{wx}}{b_y}$	$\frac{t_{wy}}{b_x}$
1	2.06	0.182	0.090	0.136	0.023	0.023
2	3.28	0.236	0.125	0.140	0.031	0.023
3	3.48	0.155	0.238	0.246	0.055	0.041
4	3.50	0.143	0.292	0.288	0.073	0.048
5	2.81	0.248	0.110	0.151	0.027	0.025
6	2.92	0.231	0.271	0.336	0.068	0.056
7	3.36	0.300	0.129	0.163	0.032	0.027
8	2.66	0.290	0.166	0.199	0.041	0.033
9	2.84	0.280	0.100	0.144	0.025	0.024
10	2.90	0.149	0.072	0.045	0.018	0.007

[a]Note: Spanwise stiffener spacing, $b_y = 4$ in. Chordwise stiffener spacing, $b_x = 6$ in.

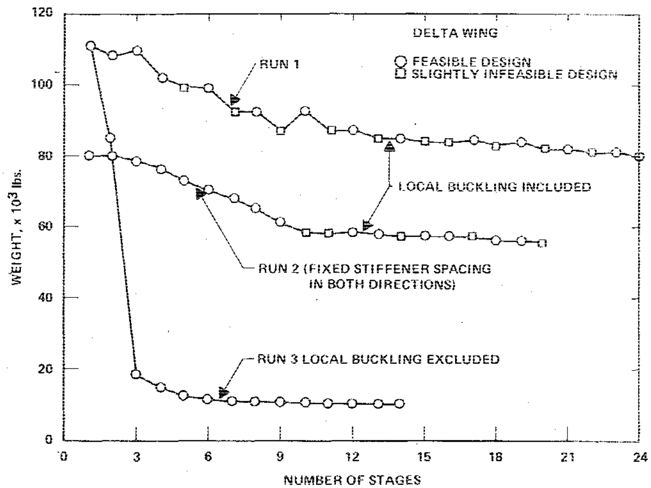

Fig. 7 Iteration history for the delta wing.

component level for waffle plates. It is thought that fixing the stiffener spacing tends to shift the component designs from one relative minimum pocket to another. The final component level results for run 2 are summarized in Table 3. It is found that both gross panel and spanwise local stiffener buckling tend to be critical at the final design particularly in regions 2, 7, 9, and 10 while panels in regions 3, 4, and 5 are critical in gross panel buckling. It is interesting to note that H, the total panel depth, tends to approach its upper bound of 3.5 in. in heavily loaded regions such as 3, 4, and 7 (see Fig. 6).

A final run (No. 3) was made in which the delta wing was designed ignoring local buckling. This design had a weight of 10,454 lb. This low weight can be attributed to the fact that the fixed allowable stresses are much larger than the allowable buckling stresses obtained in runs 1 and 2. The iteration histories for the three runs are shown in Fig. 7.

Conclusions

This paper presents a rational multilevel approach to the minimum weight optimum design of structures subject to local and system buckling constraints. The previous shortcomings of the multilevel approach are overcome a) by employing change of stiffness as the component level objective function and b) by using approximation concepts at the system level. Mathematical programming methods are used at both the component and system levels. In addition to sizing, stress, and displacement constraints it is now possible also to guard against local and system level buckling constraints. The multilevel method presented may be viewed as a decomposition technique in which the formulation is guided by physical insight. While numerical examples for truss and wing box structures are presented here, the method is potentially applicable to a broader class of problems. In particular, it should be noted that the work reported here could be extended to systems involving various types of stiffened metal or fiber composite panels as components.

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